

## EXTENSION DIMENSIONAL APPROXIMATION THEOREM

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ABSTRACT. It is known that if an upper semicontinuous multivalued mapping  $F: X \rightarrow Y$ , defined on an  $n$ -dimensional compactum  $X$ , has  $UV^{n-1}$ -point images, then every neighbourhood of the graph of  $F$  (in the product  $X \times Y$ ) contains the graph of a single-valued continuous mapping  $f: X \rightarrow Y$ . Similar result is known to be true when  $X$  is a compact  $C$ -space and images of  $F$  have trivial shape. We extend and unify both of these results in terms of extension theory.

## 1. INTRODUCTION

Single-valued approximations of multivalued maps are proved to be very useful in geometric topology, fixed point theory, control theory and others (see a survey [7]). We consider the problem of single-valued continuous graph-approximation of upper semicontinuous (u.s.c.) multivalued mappings. We say that a multivalued mapping  $F: X \rightarrow Y$  admits graph-approximations if every neighborhood of the graph of  $F$  (in the product  $X \times Y$ ) contains the graph of a single-valued continuous mapping  $f: X \rightarrow Y$ .

Essentially there are three types of results concerning our problem. First assumes that multivalued mappings  $F: X \rightarrow Y$  have  $UV^{n-1}$  point-images and  $\dim X \leq n$  (see [9, 10, 8]). The second type of results deal with  $UV^\infty$ -valued mappings defined on  $C$ -spaces [1]. Finally results of the third type consider  $UV^\infty$ -valued mappings defined on  $ANR$ -spaces [6, 8].

In this paper we prove an approximation theorem which generalizes and unifies the known results of the first and second types. Unification is achieved by exploiting recently created [4], [5] theory of extension dimension and associated to it concepts of homotopy and shape [2]. Precise definitions will be given below in Section 2. Here we only provide some of the notation related to the extension dimension.

Let  $L$  be a CW-complex. A space  $X$  is said to have *extension dimension*  $\leq [L]$  (notation:  $\text{e-dim} X \leq [L]$ ) if any mapping of its closed subspace  $A \subset X$

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into  $L$  admits an extension to the whole space  $X^1$ . It is known that  $\dim X \leq n$  is equivalent to  $\text{e-dim} X \leq [S^n]$  and that  $\dim_G X \leq n$  is equivalent to  $\text{e-dim} X \leq [K(G, n)]$  ( $K(G, n)$  stands for the corresponding Eilenberg-MacLane complex). One can develop homotopy and shape theories specifically designed to work for at most  $[L]$ -dimensional spaces. Compacta of trivial  $[L]$ -shape are precisely  $UV^{[L]}$ -compacta [2].

Now we are ready to formulate our main result.

**Theorem.** *Let  $L$  be a countable CW-complex and  $F: X \rightarrow Y$  be an u.s.c.  $UV^{[L]}$ -valued mapping of a paracompact space  $X$  to a completely metrizable space  $Y$ . If  $X$  is C-space of extension dimension  $\text{e-dim} X \leq [L]$ , then every neighborhood of the graph of  $F$  contains the graph of a single-valued continuous mapping  $f: X \rightarrow Y$ .*

Note that if  $L$  is the sphere  $S^n$ , we obtain an approximation theorem for  $UV^{n-1}$ -valued mappings of  $n$ -dimensional space. And if  $L$  is a point (or any other contractible complex), we obtain a theorem of Ancel on approximations of  $UV^\infty$ -valued mappings of C-space [1].

What do we need to construct a mapping from a space  $X$ ? Suppose that we can construct and, moreover, extend a mapping from  $X$  locally. Then one can try to obtain a fine cover of  $X$  and to construct a global mapping by induction, extending it successively over "skeleta" of this cover. The problem is to control this process when the cover has infinite order. Property C gives us a possibility of such a control.

Let us explain this with a bit more detail. A topological space  $X$  has *property C* if for each sequence  $\{u^i \mid i \geq 1\}$  of open covers of  $X$ , there is an open cover  $\Sigma$  of  $X$  of the form  $\cup_{i=1}^\infty \sigma_i$  such that for each  $i \geq 1$ ,  $\sigma_i$  is a pairwise disjoint collection which refines  $u^i$ . If the space  $X$  is paracompact, we can choose the cover  $\Sigma$  to be locally finite. The cover  $\Sigma$  has very important property that every "simplex"  $\{s_0, \dots, s_n\}$  of this cover (i.e. the set of elements  $\{s_0, \dots, s_n\}$  such that  $s_0 \cap \dots \cap s_n \neq \emptyset$ ) has a natural order on its vertices. Indeed, for any element  $s \in \Sigma$  denote by  $\sigma(s)$  the integer such that  $s \in \sigma_{\sigma(s)}$ . Since  $s_i \cap s_j \neq \emptyset$ , then  $\sigma(s_i) \neq \sigma(s_j)$  and we can order elements  $s_0, s_1, \dots, s_n$  according to the order of numbers  $\sigma(s_0), \sigma(s_1), \dots, \sigma(s_n)$ .

We take a cover  $\Sigma$  of  $X$  which refines our fine cover so that every simplex  $\langle \sigma_0, \dots, \sigma_n \rangle$  of the nerve  $N(\Sigma)$  has a natural order on its vertices. Then every simplex has a *basic* vertex (merely the smallest one). For every vertex  $\langle \sigma \rangle$  of  $N(\Sigma)$  (i.e. for every element  $\sigma$  of the cover  $\Sigma$ ) we fix a "rule" of extension of mappings defined on subsets of  $\sigma$ . Then the process goes on by induction on dimension of "skeleta" as follows: for a "simplex" the extension of mapping from the "boundary" to the "interior" is induced by the rule of the basic vertex

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<sup>1</sup>Everywhere below  $[L]$  denotes the class of complexes generated by  $L$  with respect to the above extension property, see [4], [5], [2] for details.

of this simplex. Obviously the mapping on a simplex depends only on the basic vertex of this simplex and does not depend on the dimension of the simplex. This provides the needed control.

## 2. PRELIMINARIES

Let us recall some definitions and introduce our notations. We denote by  $\text{Int}A$  and  $\overline{A}$  the interior and closure of the set  $A$  respectively. For a cover  $\omega$  of a space  $X$  and for a subset  $A \subseteq X$  let  $\text{St}(A, \omega)$  denote the star of the set  $A$  with respect to  $\omega$ .

The *graph* of a multivalued mapping  $F: X \rightarrow Y$  is the subset  $\Gamma_F = \{(x, y) \in X \times Y: y \in F(x)\}$  of the product  $X \times Y$ . A multivalued mapping  $F: X \rightarrow Y$  is called *upper semicontinuous* (notation: u.s.c.) if for any open set  $U \subset Y$  the set  $\{x \in X: F(x) \subset U\}$  is open in  $X$ .

Let  $L$  be a CW-complex. A pair of spaces  $V \subset U$  is said to be  $[L]$ -connected if for every paracompact space  $X$  of extension dimension  $\text{e-dim}X \leq [L]$  and for every closed subspace  $A \subset X$  any mapping of  $A$  into  $V$  can be extended to a mapping of  $X$  into  $U$ . A compact subspace  $K \subset Z$  is called  $UV^{[L]}$ -compactum in  $Z$  if any neighborhood  $U$  of  $K$  contains a neighborhood  $V$  of  $K$  such that the pair  $V \subset U$  is  $L$ -connected. A compact-valued mapping  $F: X \rightarrow Y$  is called  $UV^{[L]}$ -valued if for any point  $x \in X$  the set  $F(x)$  is  $UV^{[L]}$ -compactum in  $Y$ . A mapping  $f: Y \rightarrow X$  is said to be  $[L]$ -soft if for any paracompact space  $Z$  with  $\text{e-dim}Z \leq [L]$ , its closed subspace  $A \subset Z$  and any mappings  $g: Z \rightarrow X$  and  $\tilde{g}_A: A \rightarrow Y$  such that  $f \circ \tilde{g}_A = g|_A$  there exists a mapping  $\tilde{g}: Z \rightarrow Y$  such that  $f \circ \tilde{g} = g$ . Finally let  $AE([L])$  denote the class of spaces with  $[L]$ -soft constant mappings.

Now we introduce the notion of  $[L]$ -extension which will represent a "rule" for extending mappings in the proof of our theorem. Let  $V \subset U$  be a pair of spaces. An  $[L]$ -extension of the space  $V$  with respect to  $U$  is a pair  $V' \subset W$  of spaces and a mapping  $e: W \rightarrow U$  such that:

- 1)  $W \in AE([L])$ ;
- 2)  $e|_{V'}$  is  $[L]$ -soft mapping onto  $V$ .

The following is a key property of  $[L]$ -extensions needed in the proof (Section 3) of our theorem. Let a pair  $V' \subset W$  of spaces and a mapping  $e: W \rightarrow U$  represent an  $[L]$ -extension of the pair  $V \subset U$ .

**$[L]$ -extension property.** *Let  $A \subset B$  be a pair of closed subspaces of paracompact space  $X$  of extension dimension  $\text{e-dim}X \leq [L]$ . Suppose that we have mappings  $f: B \rightarrow U$  and  $g: A \rightarrow W$  such that  $e \circ g = f|_A$ ,  $f(\overline{B \setminus A}) \subset V$  and  $g(A \cap \overline{B \setminus A}) \subset V'$ . Then there exists a mapping  $g': X \rightarrow W$  such that  $e \circ g'|_B = f$ .*

We construct  $g'$  in two steps. First, we use  $[L]$ -softness of  $e$  over  $V$  to extend  $g$  to a mapping  $\tilde{g}: B \rightarrow W$  such that  $e \circ \tilde{g} = f$  (we apply  $[L]$ -softness to the  $[L]$ -dimensional pair  $A \cap \overline{B \setminus A} \subset \overline{B \setminus A}$ ). Finally we can extend  $\tilde{g}$  to the space  $X$  since  $W$  is  $AE([L])$ .

**Lemma.** *Let  $V \subset U$  be  $[L]$ -connected pair. If  $V$  is completely metrizable space, then  $V$  admits an  $[L]$ -extension with respect to  $U$ .*

*Proof.* There exists a completely metrizable space  $V'$  with  $\text{e-dim} V' \leq [L]$  and an  $[L]$ -soft mapping  $e_V: V' \rightarrow V$  [3]. Consider an  $AE([L])$ -space  $W$  of dimension  $\text{e-dim} W \leq [L]$  containing  $V'$  as a closed subspace [3]. Since the pair  $V \subset U$  is  $[L]$ -connected, we can extend the mapping  $e_V$  to a mapping  $e: W \rightarrow U$ .  $\square$

### 3. PROOF OF THE THEOREM

For a given  $UV^{[L]}$ -valued mapping  $F: X \rightarrow Y$  we fix an arbitrary neighborhood  $\mathcal{U} \subset X \times Y$  of its graph  $\Gamma_F$ . The proof of our theorem consists of the following two steps.

#### 1. Construction of families of rectangles.

For every integer  $i \geq 0$  we construct families of open rectangles  $\{u_\lambda^i \times U_\lambda^i\}_{\lambda \in \Lambda_i}$  and closed rectangles  $\{v_\mu^i \times V_\mu^i\}_{\mu \in M_i}$  in the product  $X \times Y$  such that:

- (1)  $u_\lambda^0 \times U_\lambda^0 \subset \mathcal{U}$  for every  $\lambda \in \Lambda_0$ ;
- (2)  $u^i = \{u_\lambda^i\}_{\lambda \in \Lambda_i}$  and  $v^i = \{v_\mu^i\}_{\mu \in M_i}$  are coverings of  $X$  (in fact,  $\{\text{Int } v_\mu^i\}_{\mu \in M_i}$  are coverings of  $X$ );
- (3)  $F(u_\lambda^i) \subset U_\lambda^i$  and  $F(v_\mu^i) \subset \text{Int } V_\mu^i$  for every  $i \geq 0$ ,  $\mu \in M_i$  and  $\lambda \in \Lambda_i$ ;
- (4) for every  $i \geq 0$  and every  $\mu \in M_i$  there exists  $\lambda \in \Lambda_i$  such that  $V_\mu^i \subset U_\lambda^i$ ,  $v_\mu^i \subset u_\lambda^i$ , and the pair  $V_\mu^i \subset U_\lambda^i$  is  $[L]$ -connected;

*Choice 1.* For given  $i \geq 0$  and  $\mu \in M_i$  we fix such a  $\lambda = \lambda(\mu)$ , and for  $[L]$ -connected pair  $V_\mu^i \subset U_\lambda^i$ , by Lemma, we can fix  $[L]$ -extension  $e_\mu^i: (\tilde{V}_\mu^i, W_\mu^i) \rightarrow (V_\mu^i, U_\lambda^i)$ ;

- (5) for every  $i \geq 0$  and every  $\lambda \in \Lambda_{i+1}$  there exists  $\mu \in M_i$  such that  $\text{St}(u_\lambda^{i+1}, u^{i+1}) \subset v_\mu^i$  and every rectangle  $u_\gamma^{i+1} \times U_\gamma^{i+1}$  is contained in the rectangle  $v_\mu^i \times V_\mu^i$  provided  $u_\gamma^{i+1} \cap u_\lambda^{i+1} \neq \emptyset$ ;

*Choice 2.* For given  $i \geq 0$  and  $\lambda \in \Lambda_{i+1}$  we fix such a  $\mu = \mu(\lambda)$ .

First, we construct a family  $\{u_\lambda^0 \times U_\lambda^0\}_{\lambda \in \Lambda_0}$ . Put  $\Lambda_0 = X$  and for a point  $x \in X$  consider a rectangle  $u_x \times U_x^0 \subset \mathcal{U}$  such that  $F(x) \subset U_x^0$  (existence of such a rectangle follows from compactness of  $F(x)$ ). Since  $F$  is u.s.c., we can choose a neighborhood  $u_x^0 \subset u_x$  of the point  $x$  such that  $F(u_x^0) \subset U_x^0$ .

The construction of families of rectangles is performed by induction on  $i$ . All steps of induction are similar to the first one. Here we only show how to perform the first step and to construct the families  $\{v_\mu^0 \times V_\mu^0\}_{\mu \in M_0}$  and  $\{u_\lambda^1 \times U_\lambda^1\}_{\lambda \in \Lambda_1}$ .

Put  $M_0 = X$  and for a point  $x \in X$  consider a rectangle  $u_\lambda^0 \times U_\lambda^0$  containing  $\{x\} \times F(x)$ . By  $UV^{[L]}$ -property of  $F(x)$  we find a closed neighborhood  $V_x^0$  of  $F(x)$  such that the pair  $V_x^0 \subset U_\lambda^0$  is  $[L]$ -connected. Since  $F$  is u.s.c., we can choose a closed neighborhood  $v_x^0 \subset u_\lambda^0$  of the point  $x$  such that  $F(v_x^0) \subset \text{Int}V_x^0$ .

Now we construct a family  $\{u_\lambda^1 \times U_\lambda^1\}_{\lambda \in \Lambda_1}$ . Let  $\alpha$  be a locally finite open cover of  $X$  refining  $v^0$ . For every element  $A \in \alpha$  take an index  $\mu \in M_0$  such that  $A \subset v_\mu^0$  and denote  $W_A = \text{Int}V_\mu^0$ . Then  $A \times W_A$  lies in  $v_\mu^0 \times V_\mu^0$ . Let  $u^1 = \{u_\lambda^1\}_{\lambda \in \Lambda_1}$  be an open cover of  $X$  which is star-refined into  $\alpha$ . Define

$$U_\lambda^1 = \bigcap \{W_A \mid \text{St}(u_\lambda^1, u^1) \subset A \in \alpha\}.$$

To verify (5), consider  $u_{\lambda'}^1 \in u^1$  such that  $u_{\lambda'}^1 \cap u_\lambda^1 \neq \emptyset$ . Then  $u_{\lambda'}^1 \subset \text{St}(u_\lambda^1, u^1) \subset A$  for some  $A \in \alpha$  and by definition  $U_{\lambda'}^1 \subset W_A$ . Thus,  $u_{\lambda'}^1 \times U_{\lambda'}^1 \subset A \times W_A \subset v_\mu^0 \times V_\mu^0$ .

## 2. Construction of the map $f$ .

Since  $X$  is a paracompact  $C$ -space, there exists a locally finite open cover  $\Sigma$  of  $X$  of the form  $\Sigma = \bigcup_{i=1}^\infty \sigma_i$  such that for  $i \geq 1$ ,  $\sigma_i$  is pairwise disjoint collection refining  $u^i$ . For every integer  $k \geq 0$  denote by  $\Sigma^{(k)}$  the set of points  $x \in X$  such that the cover  $\Sigma$  has order  $\leq k+1$  at  $x$ . Note that  $X = \bigcup_{i=0}^\infty \Sigma^{(k)}$  and  $\Sigma^{(k)}$  is closed in  $X$ . We will construct  $f$  inductively extending it over sets  $\Sigma^{(k)}$ .

For any element  $s$  of the cover  $\Sigma$  we denote by  $\sigma(s)$  the integer number such that  $s \in \sigma_{\sigma(s)}$ .

*Choice 3:* for any element  $s \in \Sigma$  we fix  $\lambda(s) \in \Lambda_{\sigma(s)}$  such that  $s \subset u_{\lambda(s)}^{\sigma(s)}$ .

Let  $s_0, s_1, \dots, s_n$  be elements of the cover  $\Sigma$  such that  $s_0 \cap s_1 \cap \dots \cap s_n \neq \emptyset$ . Then this set of elements could be ordered according to the order of numbers  $\sigma(s_0), \sigma(s_1), \dots, \sigma(s_n)$ , and the smallest element of the set  $\{s_0, s_1, \dots, s_n\}$  is called the *basic element*. We always assume that  $s_0$  is the basic element of the set  $\{s_0, s_1, \dots, s_n\}$ . We will use the following notations

$$[s_0, s_1, \dots, s_n] = X \setminus \bigcup \{\Sigma \setminus \{s_0, s_1, \dots, s_n\}\}$$

$$\langle s_0, s_1, \dots, s_n \rangle = (s_0 \cap s_1 \cap \dots \cap s_n) \cap \Sigma^{(n)}.$$

One should understand the set  $[s_0, \dots, s_n]$  as closed  $n$ -dimensional "simplex" with interior  $\langle s_0, s_1, \dots, s_n \rangle$  and boundary  $\bigcup_{m=0}^n [s_0, s_1, \dots, \widehat{s_m}, \dots, s_n]$ . It is easy to check that  $\Sigma^{(n)} = \bigcup [s_{i_0}, s_{i_1}, \dots, s_{i_n}]$  and

$$[s_0, \dots, s_n] = \bigcup_{m=0}^n [s_0, \dots, \widehat{s_m}, \dots, s_n] \cup \langle s_0, \dots, s_n \rangle.$$

Let us construct the mapping  $f$  on the set  $\Sigma^{(0)}$  which is a discrete collection of sets of the type  $[s_0]$ . We define  $f$  independently on every such a set. For a set  $[s_0]$  we take a point  $p \in F([s_0])$  and put  $f([s_0]) = p$ .

Let us extend  $f$  to arbitrary nonempty set  $\langle s_0, s_1 \rangle$ . For  $i = 0, 1$  we have  $\langle s_i \rangle \subset u_{\lambda(s_i)}^{\sigma(s_i)}$  and then  $f(\langle s_i \rangle) \subset U_{\lambda(s_i)}^{\sigma(s_i)}$  by property (3). According to the choice 2, we take  $\mu \in M_{\sigma(s_0)-1}$  such that

$$[s_0, s_1] \subset \overline{\text{St}(u_{\lambda(s_0)}^{\sigma(s_0)}, u_{\lambda(s_0)}^{\sigma(s_0)})} \subset v_{\mu}^{\sigma(s_0)-1} \quad \text{and} \quad f([s_0]) \cup f([s_1]) \subset V_{\mu}^{\sigma(s_0)-1}.$$

Choice 1 gives us  $\lambda = \lambda(\mu)$ , a set  $U_{\lambda}^{\sigma(s_0)-1}$  and  $[L]$ -extension

$$e_{\mu}^{\sigma(s_0)-1}: \left( \tilde{V}_{\mu}^{\sigma(s_0)-1}, W_{\mu}^{\sigma(s_0)-1} \right) \rightarrow \left( V_{\mu}^{\sigma(s_0)-1}, U_{\lambda}^{\sigma(s_0)-1} \right).$$

Since the mapping  $e_{\mu}^{\sigma(s_0)-1}|_{\tilde{V}_{\mu}^{\sigma(s_0)-1}}$  is  $[L]$ -soft, we can lift the map  $f|_{[s_0] \cup [s_1]}: [s_0] \cup [s_1] \rightarrow V_{\mu}^{\sigma(s_0)-1}$  to a map  $g: [s_0] \cup [s_1] \rightarrow \tilde{V}_{\mu}^{\sigma(s_0)-1}$ . Now extend  $g$  to a mapping  $\tilde{g}: [s_0, s_1] \rightarrow W_{\mu}^{\sigma(s_0)-1}$  and define  $f|_{[s_0, s_1]}$  as  $e_{\mu}^{\sigma(s_0)-1} \circ \tilde{g}$ .

We can continue our construction so that the extension to a set  $\langle s_0, s_1, \dots, s_m \rangle$  uses  $[L]$ -extension  $e_{\mu}^{\sigma(s_0)-1}$  and goes through  $W_{\mu}^{\sigma(s_0)-1}$  resulting as  $f|_{[s_0, \dots, s_m]} = e_{\mu}^{\sigma(s_0)-1} \circ \tilde{g}$ . Therefore, the set  $f([s_0, \dots, s_m])$  is contained in  $U_{\lambda}^{\sigma(s_0)-1}$  while the set  $[s_0, \dots, s_m]$  lies in  $u_{\lambda}^{\sigma(s_0)-1}$ . Note that both indices  $\lambda$  and  $\mu$  depend only on the basic element  $s_0$  and do not depend on  $m$ . So,  $[L]$ -extension  $e_{\mu}^{\sigma(s_0)-1}$  is a "rule" for constructing mapping on each set  $\langle s_0, \dots, s_m \rangle$  with basic element  $s_0$ .

Suppose that the map  $f$  is constructed on  $\Sigma^{(k-1)}$ . Let us extend  $f$  independently to every set of type  $\langle s_0, \dots, s_k \rangle$ . Since the difference  $\Sigma^{(k)} \setminus \Sigma^{(k-1)}$  is covered by a discrete family of such sets, it follows that the so obtained extension of  $f$  to  $\Sigma^{(k)}$  would be continuous. Assume that  $s_1$  is basic element of the set  $\{s_1, s_2, \dots, s_k\}$ . Then the set  $f(\langle s_1, \dots, s_k \rangle)$  lies in some  $U_{\lambda_1}^{\sigma(s_1)-1}$  and  $u_{\lambda_1}^{\sigma(s_1)-1}$  contains  $[s_1, \dots, s_k]$ . Since  $\sigma(s_1) - 1 \geq \sigma(s_0)$ , the set  $f(\langle s_1, \dots, s_k \rangle)$  lies in  $V_{\mu}^{\sigma(s_0)-1}$  by property (5). Let

$$G = \bigcup_{1 \leq m \leq k} [s_0, s_1, \dots, \hat{s}_m, \dots, s_k].$$

Then, by our construction,  $f|_G$  has a lift  $g: G \rightarrow W_{\mu}^{\sigma(s_0)-1}$ . Note that

$$f(\overline{\langle s_1, \dots, s_k \rangle} \cap G) \subseteq V_{\mu}^{\sigma(s_0)-1} = \overline{V_{\mu}^{\sigma(s_0)-1}}.$$

Since the mapping  $e_\mu^{\sigma(s_0)-1} : \tilde{V}_\mu^{\sigma(s_0)-1} \rightarrow V_\mu^{\sigma(s_0)-1}$  is  $[L]$ -soft, we extend the lift  $g$  to the set  $\langle s_1, \dots, s_k \rangle$ . Now extend it to a mapping  $g : [s_0, \dots, s_k] \rightarrow W_\mu^{\sigma(s_0)-1}$  and define  $f|_{[s_0, \dots, s_k]}$  as the composition  $e_\mu^{\sigma(s_0)-1} \circ g$ .

It only remains to note that the local finiteness of  $\Sigma$  guarantees the continuity of the above constructed map  $f$ . Proof is completed.

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## REFERENCES

- [1] F. D. Ancel, *The role of countable dimensionality in the theory of cell-like embedding relations*, Trans. Amer. Math. Soc. **287** (1985) 1–40.
- [2] A. Chigogidze, *Infinite Dimensional Topology and Shape Theory*, in: Handbook of Geometric Topology, ed. by R. Daverman and R. Sher (to appear).
- [3] A. Chigogidze, V. Valov, *Universal metric spaces and extension dimension*, Topology Appl., **113** (2001), 23–27.
- [4] A. N. Dranishnikov, *The Eilenberg-Borsuk theorem for maps in an arbitrary complex*, Math. Sbornik, **185** (1994), no. 4, 81–90 (in Russian); translation in: Russian Acad. Sci. Sb. Math. **81** (1995), no. 2, 467–475.
- [5] A. N. Dranishnikov, J. Dydak, *Extension dimension and extension types*, Tr. Mat. Inst. Steklova **212** (1996), Otobrazh. i Razmer., 61–94.
- [6] L. Gorniewicz, A. Granas, W. Kryszewski, *On the homotopy method in the fixed point index theory of multi-valued mappings of compact absolute neighborhood retracts*, J. Math. Anal. Appl. **161** (1991), 457–473.
- [7] W. Kryszewski, *Graph-approximation of set-valued maps. A survey*, Differential Inclusions and Optimal Control. Lecture Notes in Nonlinear Analysis **2** (1998), 223–235.
- [8] W. Kryszewski, *Graph-approximation of set-valued maps on noncompact domains*, Topology Appl. **83** (1998), 1–21.
- [9] R.C. Lacher, *Cell-like mappings and their generalizations*, Bull. AMS **83** (1977), 495–552.
- [10] E.V. Ščepin, N.B. Brodsky, *Selections of filtered multivalued mappings*, Tr. Mat. Inst. Steklova **212** (1996), Otobrazh. i Razmer., 220–240.

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